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1994 J. Phys.: Condens. Matter 6 L583

(http://iopscience.iop.org/0953-8984/6/39/002)

J. Phys.: Condens. Matter 6 (1994) L583–L588. Printed in the UK

LETTER TO THE EDITOR

Quantum transport in ballistic nano-scale Corbino disks

George Kirczenow
Department of Physics, Simon Fraser University, Burnaby, British Columbia V5A 1S6, Canada

Received 8 August 1994

Abstract. A theoretical study is presented of transport through ballistic nano-scale Corbino disks with ideal contacts. Quantum, classical and semi-classical results are compared. At magnetic field $B = 0$ and low temperatures, the conductance G is quantized in *odd* integer multiples of $2e^2/h$. G is not a monotonic function of B, because conduction via successive quantum states switches on and off as B increases. G shows an approximate plateau at low B (for $r_c > (r_i + r_o)/2$) and is zero at high B, for $r_c < (r_o - r_i)/2$. r_c is the cyclotron radius. r_i and r_o are the radii of the inner and outer contacts. In between, G falls approximately linearly with B, modulated by structures on the scale of e^2/h due to conduction through individual quantum states.

A Corbino disk is an annular region of conducting material (for example, a two-dimensional electron gas (2DEG) in a semiconductor heterostructure) surrounding a metallic contact and surrounded in turn by a second metallic contact. This is illustrated in the inset of figure 1, where the inner and outer metallic contacts are labelled I and O. Macroscopic Corbino disks played an important role in the thought experiments [1, 2] that first clarified the nature of the integer quantum Hall effect [3]. A unique feature of Corbino disks is that, unlike other two-dimensional semiconductor devices, their boundaries consist entirely of metallic contacts. Thus in a magnetic field they do not exhibit edge states of the usual type [2]. The physics of the macroscopic Corbino disks considered theoretically to date has been strongly influenced by the presence of defects [1, 2]. However, it is reasonable to expect that *nano-scale* Corbino disks will be ballistic, effectively defect-free systems, in common with other semiconductor nanostructures [4]. Such nano-scale Corbino disks have not as yet been studied experimentally or theoretically, although Corbino disks a few microns in size have recently been fabricated [5].

In this letter, the first theoretical study of the transport properties of ballistic nano-scale Corbino disks with ideal contacts is presented. It is hoped that this work will stimulate interest in these novel systems and facilitate future experiments. It is predicted that in the absence of magnetic fields, ballistic Corbino disks should exhibit conductance quantization in *odd* integer multiples of $2e^2/h$. This is analogous to the conductance quantization observed in ballistic point contacts [4, 6]. However, in point contacts, *all* integer multiples of $2e^2/h$ are seen. Classically, the conductance of a ballistic Corbino disk, at fixed Fermi energy, remains constant as the magnetic field increases from zero, until the cyclotron radius r_c becomes equal to $(r_o + r_i)/2$, the average of the radii of the inner and outer contacts r_i and r_o (see the inset of figure 1). As the magnetic field increases further, the conductance begins to decrease, reaching zero when $r_c = (r_o - r_i)/2$ and electrons emitted from one contact can no longer reach the other. The results of quantum mechanical calculations follow

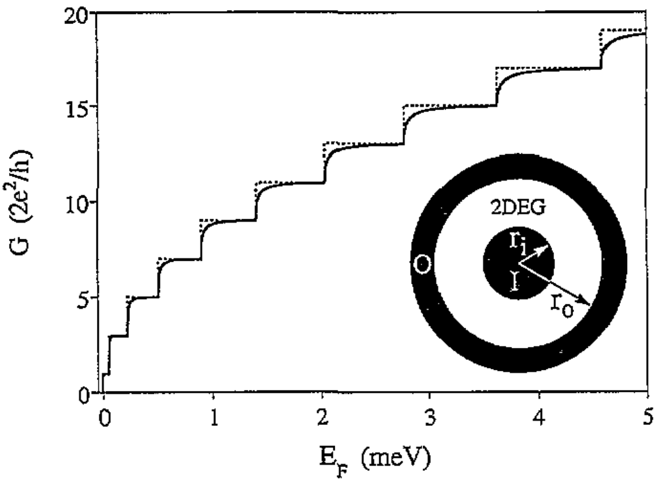

Figure 1. Conductance G of a ballistic Corbino disk with ideal contacts versus Fermi energy E_F at $B = 0$ and zero temperature for $r_i = 0.1\mu$, $r_o = 0.2\mu$ and $V(r) = 0$. Solid line is the quantum conductance, dotted line is the result of semi-classical approximation. Inset: schematic of Corbino disk with inner and outer radii r_i and r_o. Inner and outer metallic contacts are labelled I and O respectively.

the classical behaviour, but with interesting differences: at low temperatures, the quantum conductance does not vary monotonically with magnetic field at fixed Fermi energy, but exhibits structure due to the switching on and off of conduction through a succession of different quantum states as the magnetic field increases.

An electron in a magnetic field obeys the Schrödinger equation

$$\left(\frac{1}{2m^*}\left(\frac{\hbar}{\mathrm{i}}\nabla + eA\right)^2 + V\right)\Psi = E\Psi \tag{1}$$

where A is the vector potential, V is the electron's electrostatic potential energy and m^* is its effective mass. For azimuthal symmetry and a uniform magnetic field B described by the vector potential $A = (-By/2, Bx/2, 0)$, the eigenfunctions for an electron in two dimensions take the form

$$\Psi_l(r, \theta) = \mathrm{e}^{\mathrm{i}l\theta} g_l(r) r^{-1/2} \tag{2}$$

and the Schrödinger equation reduces to

$$-\frac{\partial^2}{\partial r^2} g_l(r) + \left(\frac{1}{r^2}\left(l^2 - \frac{1}{4}\right) + \frac{r^2}{4a^2} + \frac{l}{a^2} + \tilde{V}(r)\right) g_l(r) = \tilde{E} g_l(r) \tag{3}$$

where $a = [h/(2\pi Be)]^{1/2}$ is the magnetic length, $\tilde{E} = 2m^*E/\hbar^2$, $\tilde{V} = 2m^*V/\hbar^2$ and l is any integer.

The above theory applies to non-interacting electrons in the annular 2DEG between the metallic contacts. In order to study electron transport through the Corbino disk, however, it is also necessary to model the emission and absorption of electrons by the contacts [7]. In this letter, a model of ideal contacts is adopted. The model is constructed so that each mode, l, of the Corbino disk flows freely into and out of the contacts and any electron that enters a contact is absorbed by it. This is achieved by treating each contact as if it were a two-dimensional system with an l-dependent effective potential energy function $U_l(r)$ and defining $\tilde{U} = 2m^*U/\hbar^2$. For the outer (inner) contact O (I) where $r > r_\text{o}$ ($r < r_\text{i}$), $U_l(r)$ is chosen so that

$$\frac{1}{r^2}\left(l^2-\frac{1}{4}\right)+\frac{r^2}{4a^2}+\frac{l}{a^2}+\tilde{U}_l(r)=\frac{1}{r_x^2}\left(l^2-\frac{1}{4}\right)+\frac{r_x^2}{4a^2}+\frac{l}{a^2}+\tilde{V}(r_x)\equiv c_x \tag{4}$$

where x stands for o (or i). With this choice [8] of U, the solutions $g_l(r)$ of the Schrödinger equation in the contact regions are of the form $g_l(r) = A_x^+ e^{ik_x r} + A_x^- e^{-ik_x r}$ where $k_x = \sqrt{\tilde{E} - c_x}$. That is, the solutions are linear combinations of incoming or outgoing radial waves. In the Landauer description of transport [9], the two-terminal conductance G of the Corbino disk is then given by

$$G = \frac{e^2}{h}\sum_{l,s} T_l \tag{5}$$

where T_l is the transmission probability of mode l through the Corbino disk from one contact to the other, and the sum is over the azimuthal modes l and values of spin index s at the Fermi energy. T_l defined in this way can be calculated by solving the differential equation (3) using standard numerical techniques. For example, for the mode l transmitted through the Corbino disk from the inner to the outer contact, one can choose the solution in the outer contact (for $r > r_\text{o}$) to be $g_l(r) = e^{ik_\text{o}r}$. Then integrating equation (3) from r_o to r_i, one obtains the transmission probability of mode l from contact to contact as

$$T_l = 4k_\text{o}k_\text{i}\Big/\left(\left(k_\text{i}g_l^*(r)+\text{i}\frac{\partial}{\partial r}g_l^*(r)\right)\left(k_\text{i}g_l^*(r)-\text{i}\frac{\partial}{\partial r}g_l(r)\right)\right)\Bigg|_{r=r_\text{i}}. \tag{6}$$

Representative results for the conductance G of a ballistic Corbino disk calculated in the above way at $B = 0$ and zero temperature as a function of the Fermi energy E_F are presented in figure 1. The solid line is the calculated conductance for a disk with $r_\text{i} = 0.1\mu$, $r_\text{o} = 0.2\mu$, and a flat electrostatic potential $V(r) = 0$. The electron effective mass is that of GaAs ($m^* = 0.067m_0$). The calculated conductance exhibits a series of plateaus near odd integer multiples of $2e^2/h$, qualitatively resembling the conductance plateaus observed in ballistic point contacts [4, 6]. The accuracy of the quantization is a few percent of e^2/h.

One can understand the conductance quantization semi-classically as follows: in the absence of magnetic fields and for $V(r) = 0$, the radial part of the Schrödinger equation (3) becomes

$$-\frac{\partial^2}{\partial r^2}g_l(r)+\frac{1}{r^2}\left(l^2-\frac{1}{4}\right)g_l(r)=\tilde{E}g_l(r). \tag{7}$$

This is a one-dimensional Schrödinger equation with an effective potential energy term $(l^2 - 1/4)/r^2$. If electron propagation through the Corbino disk is treated classically, modes with $l > 0$ for which $\tilde{E} > (l^2 - 1/4)/r_\text{i}^2$ pass over the highest point of the effective potential barrier (at $r = r_\text{i}$) and are transmitted through the Corbino disk while other modes are reflected. In this simple picture the Landauer conductance formula (5) yields the semi-classical result

$$G_\text{SC} = \frac{2e^2}{h}(2l^* + 1) \tag{8}$$

where l^* is the largest integer that is smaller than $(8\pi E_F m^* r_i^2/h^2 + 1/4)^{1/2}$. The case near the onset of conduction by the lowest ($l = 0$) mode is slightly different. There the highest effective potential barrier is at r_o, and conduction begins at $\tilde{E}_F = -1/(4r_o^2)$. G_{SC} is shown as the dotted line in figure 1. This semi-classical conductance differs for the full quantum result mainly near the onset of conduction by each mode, where the last mode to be populated is only weakly transmitted in the quantum theory.

The conductance of a ballistic Corbino disk with $r_i = 0.1\mu$, $r_o = 0.2\mu$ and $V(r) = 0$, calculated as described earlier at constant Fermi energy $E_F = 2$ meV, is shown as a function of the magnetic field B by the solid curve in figure 2. The Zeeman splitting between spin up and spin down states is neglected because the magnetic field is rather low.

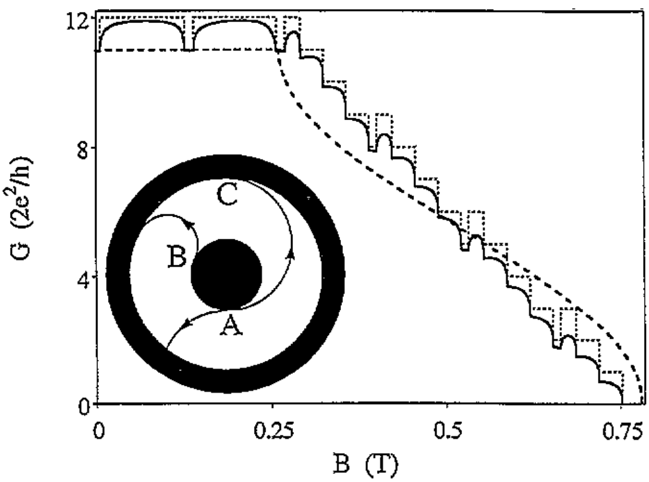

Figure 2. Conductance G versus magnetic field B, at constant Fermi energy $E_F = 2$ meV. r_i, r_o and $V(r)$ as in figure 1. Solid line is the quantum conductance, dashed line is the conductance for classical trajectories. Dotted line is the conductance in the semi-classical approximation. Inset: schematic of Corbino disk showing limiting classical electron trajectories (see text).

One can understand the broad trends in this plot in terms of a simple classical picture: suppose that each contact emits electrons isotropically with the Fermi velocity v_F, that the electrons follow classical trajectories, and that every electron that strikes a contact is absorbed. At zero magnetic field, every Fermi electron emitted by the inner contact is absorbed by the outer contact, and the Corbino disk has a conductance which will be denoted G_0. As the magnetic field increases, all of the Fermi electrons emitted by the inner contact continue to be absorbed by the outer contact, and the conductance remains constant and equal to G_0, until the cyclotron radius $r_c = m^* v_F/eB$ becomes equal to the average of the inner and outer radii of the Corbino disk, $r_c = (r_o + r_i)/2$. At that magnetic field one of the electron trajectories leaving the inner contact at a grazing angle also just grazes the outer contact. This is the trajectory between A and C in the inset of figure 2. For larger magnetic fields the trajectories of some electrons emitted by the inner contact return ballistically to the inner contact and the conductance begins to decrease. In this regime, assuming the conductance to be proportional to the fraction of emitted electrons that cross from one contact to the other, simple geometrical considerations yield the classical conductance

$$G_{\text{classical}} = G_0\left(1 - \frac{1}{\pi} a\cos\left(\frac{r_i}{2r_c}\left(1 + \frac{2r_o r_c}{r_i^2} - \frac{r_o^2}{r_i^2}\right)\right)\right) \qquad \frac{r_o + r_i}{2} > r_c > \frac{r_o - r_i}{2} \tag{9}$$

where the inverse cosine takes values in the interval $[0, \pi]$. $G_{\text{classical}}$ falls to zero (at zero temperature) when $r_c = (r_o - r_i)/2$, where the last trajectory that connects the inner and outer contacts is that denoted by B in the inset of figure 2. $G_{\text{classical}}$ is shown by the dashed line in figure 2 where the semi-classical zero field conductance G_{SC} given by equation (8) has been used for G_0. The good overall agreement between the classical and quantum results allows one to interpret the physical meaning of the gross features of the quantum conductance curve (the approximate plateau at low B ($r_c \gtrsim (r_o - r_i)/2$), and the vanishing of the conductance at higher B (for $r_c \lesssim (r_o - r_i)/2$) within this simple classical framework.

One can understand the quantum results better by extending the semi-classical picture described earlier for $B = 0$ to non-zero magnetic fields: for $B \neq 0$ (and $V(r) = 0$) the effective potential energy in the one-dimensional Schrödinger equation (3) is $\tilde{W}_l(r) = (l^2-1/4)/r^2+r^2/(4a^2)+l/a^2$. $\tilde{W}$ is not a monotonic function of r, so that one must consider the possibility of an electron in mode l encountering an effective potential barrier when passing between the 2DEG and *either* ideal contact. Making once again the semi-classical approximation that modes that pass over the potential barrier are perfectly transmitted, and others are reflected, the transmitted modes l at the Fermi energy E_F are those that obey $\tilde{E}_F \geqslant \tilde{W}_l(r_x)$ for both $x = \text{i}$ and $x = \text{o}$. This leads to the condition that for a mode l to be transmitted from contact to contact, l should satisfy $l_x^- \leqslant l \leqslant l_x^+$ for both $x = \text{i}$ and $x = 0$, where

$$l_x^{\pm} = \pm\sqrt{\tilde{E}_F r_x^2 + 1/4} - r_x^2/(2a^2). \tag{10}$$

The semi-classical conductance of the Corbino disk given by the Landauer formula is then

$$G_{\text{SC}}(B) = \frac{e^2}{h}\sum_s (l^+ - l^- + 1) \tag{11}$$

where the sum is over spin, l^+ is the largest integer smaller than both l_i^+ and l_o^+, and l^- is the smallest integer larger than both l_i^- and l_o^-. Equation (11) applies if $l^+ \geqslant l^-$; otherwise $G_{\text{SC}}(B) = 0$. $G_{\text{SC}}(B)$ is shown by the dotted line in figure 2, where the Zeeman splitting between spin up and down is neglected. $G_{\text{SC}}(B)$ changes with magnetic field in a step-wise fashion as both l^+ and l^- decrease (but not simultaneously) as the magnetic field increases. Each step is due to conduction by a particular mode l switching on or off. The conductance obtained from the full quantum calculation (the solid curve in figure 2) follows this behaviour, but is somewhat lower than $G_{\text{SC}}(B)$ and the conductance steps are more rounded because the modes that are close to being switched on or off are only partly transmitted.

At low magnetic fields both l^+ and l^- are controlled (except at $l = 0$) by the effective potential near the inner contact. From equation (10) it follows that $l_i^+ - l_i^- = (4\tilde{E}_F r_i^2 + 1)^{1/2}$ which is independent of B if the Fermi energy is held fixed. l^+ and l^- which appear in equation (11) for $G_{\text{SC}}(B)$ may differ from l_i^+ and l_i^- by at most unity. It then follows that, in this regime of low magnetic fields, $G_{\text{SC}}(B)$ should have the form of a plateau modulated by excursions of magnitude $2e^2/h$, behaviour clearly visible in figure 2 for $B < 0.25$ T.

As the magnetic field increases, eventually l_o^+ becomes smaller than l_i^+. This change occurs when

$$a^2 = \left(r_i^2 - r_o^2\right) \Big/ \left(\sqrt{4\tilde{E}_F r_i^2 + 1} - \sqrt{4\tilde{E}_F r_o^2 + 1}\right). \tag{12}$$

If the Fermi energy is large so that $\tilde{E}_F r_i^2 \gg 1$ and noting that $r_c = a^2 \tilde{E}_F^{1/2}$, equation (12) reduces to $r_c \approx (r_o + r_i)/2$. This is just where the low-B conductance plateau ends in the classical approximation as discussed above. For larger values of B, l_o^+ and l_i^- determine l^+ and l^- which appear in equation (11). From equation (10) one finds,

$$l_o^+ - l_i^- = \left(r_i^2 - r_o^2\right) / \left(2a^2\right) + \sqrt{\tilde{E}_F r_o^2 + 1/4} + \sqrt{\tilde{E}_F r_i^2 + 1/4}. \tag{13}$$

The first term on the RHS of equation (13) is negative and linear in B and the other two terms are constant if E_F is fixed. Thus in this regime if one ignores the difference between $l_o^+ - l_i^-$ and $l^+ - l^-$, one would find the semi-classical conductance $G_{SC}(B)$ given by equation (11) decreasing linearly with increasing B. This linear behaviour, modulated by steps due to the fact that l_o^+ and l^+ (and l_i^- and l^-) are not identical, is clearly visible in figure 2. Finally, again assuming that $\tilde{E}_F r_i^2 \gg 1$ one finds that $l_o^+ - l_i^- + 1 \approx 0$ when $r_c = (r_o - r_i)/2$, indicating that $G_{SC}(B)$ should vanish not far from where the classical conductance vanishes, which is also seen in figure 2.

In this letter a theory of ballistic transport in Corbino disks with ideal contacts has been presented. Quantized conductances were predicted in the absence of magnetic fields, and interesting classical and quantum behaviour described when a magnetic field is present. It is hoped that this work will stimulate interest in and experimental studies of nano-scale Corbino disks, a new class of semiconductor nanostructures.

I would like to thank A S Sachrajda, B L Johnson and P Coleridge for interesting discussions. This work was supported by the Natural Sciences and Engineering Research Council of Canada.

References

[1] Laughlin R B 1981 *Phys. Rev.* B **23** 5632

[2] Halperin B I 1982 *Phys. Rev.* B **25** 2185

[3] von Klitzing K, Dorda G and Pepper M 1980 *Phys. Rev. Lett.* 45 494

[4] See
Ulloa S E, MacKinnon A, Castaño E and Kirczenow G 1992 *Handbook on Semiconductors* vol 1, 863 for a review

[5] Sachrajda A S private communication

[6] van Wees B J, van Houten H, Beenakker C W J, Williamson J G, Kouvenhoven L P, van der Marel D and Foxon C T 1988 *Phys. Rev. Lett.* **60** 848
Wharam D A, Pepper M, Ahmed H, Frost J E F, Hasko D G, Peacock D C, Ritchie D A and Jones G A C 1989 *J. Phys. C: Solid State Phys.* **21** L209

[7] Electron transmission between metallic contacts and the 2DEG is not well understood at present. See
Hawksworth S J, Chamberlain J M, Cheng T S, Henini M, Heath M, Davies M and Page A J 1992 *Semicond. Sci. Technol.* **7** 1085
Levinson Y B 1994 *Surf. Sci.* **305** 465

[8] In this model the effective potential experienced by mode l at the boundaries between the 2DEG and the contacts is continuous. However, the coupling between the Corbino disk and the contacts is not *completely* adiabatic, since derivatives of the effective potential are not continuous.

[9] Landauer R, 1970 *Phil. Mag.* **21** 863
Fisher D A and Lee P A 1981 *Phys. Rev.* B **23** 6851
Economou E N and Soukoulis C M 1981 *Phys. Rev. Lett.* **46** 618